$\int \sin^m x \cos^n x \, dx$

Let $I_{m,n} = \int \sin^m x \cos^n x \, dx$

Where m and n are positive integers

$$I_{m,n} = \int (\sin^m x \cos x) \cos^{n-1} x \, dx \quad \text{---} (1)$$

Consider $\int \sin^m x \cos x \, dx = \int t^m dt = \frac{t^{m+1}}{m+1} = \frac{\sin^{m+1} x}{m+1}$

Applying integration by parts on RHS of equation (1), we get

$$\therefore I_{m,n} = \cos^{n-1} x \left[\frac{\sin^{m+1} x}{m+1}\right] - \int (n-1) \cos^{n-2} x (-\sin x) \left[\frac{\sin^{m+1} x}{m+1}\right] dx$$

$$= \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} \int \sin^m x . \cos^{n-2} x \sin^2 x \, dx$$

$$= \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} \int \sin^m x \cos^{n-2} x \left(1 - \cos^2 x\right) dx$$

$$= \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} \int \left(\sin^m x \cos^{n-2} x - \sin^m x \cos^n x\right) dx$$

$$= \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} \int \sin^m x \cos^{n-2} x \, dx - \frac{n-1}{m+1} \int \sin^m x \cos^n x \, dx$$

$$I_{m,n} = \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} I_{m,n-2} - \frac{n-1}{m+1} I_{m,n}$$

$$I_{m,n} + \frac{n-1}{m+1} I_{m,n} = \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} I_{m,n-2}$$

$$\left[1 + \frac{n-1}{m+1}\right] I_{m,n} = \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} I_{m,n-2}$$

$$\left[\frac{m+1+n-1}{m+1}\right] I_{m,n} = \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} I_{m,n-2}$$

$$\frac{m+n}{m+1} I_{m,n} = \frac{\cos^{n-1} x \sin^{m+1} x}{m+1} + \frac{n-1}{m+1} I_{m,n-2}$$

$$I_{m,n} = \frac{\cos^{n-1} x \sin^{m+1} x}{m+n} + \frac{n-1}{m+n} I_{m,n-2}$$

This is the reduction formula for $\int \sin^m x \cos^n x \, dx$.